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# **Price-setting mixed duopoly, subsidization and the order of firms' moves: the relevance of privatization**

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## **Abstract**

This paper first examines a price-setting mixed duopoly game with production subsidies where a public firm acts as a leader against a private firm. Second, the paper examines a price-setting duopoly game with production subsidies where the public firm remains a leader after privatization. Third, the paper compares the equilibrium values for private leadership with those for public leadership.

Keywords: Price competition; Subsidy; Privatization; Mixed Stackelberg duopoly; Privatized Stackelberg duopoly

JEL classification: C72; D21; L32

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# 1. Introduction

In recent years, the theoretical analysis of privatization of public firms has been extensively studied by many researchers. For instance, White (1996) shows three effects of production subsidies in a mixed Cournot oligopoly market regarding privatization and efficiency. First, when subsidies are used before and after privatization, privatization does not change economic welfare. Second, if subsidies are used only before privatization, then economic welfare is always lower after privatization. Third, the subsidy contributes to overall efficiency in a mixed Cournot market due to cost distribution effects. Poyago-Theotoky (2001) extends the work by White (1996) and shows that the optimal production subsidy is identical irrespective of whether (i) a public firm and  $n$  private firms simultaneously choose output, (ii) the public firm acts as a Stackelberg leader, or (iii) all firms behave as profit-maximizers. In addition, Ohnishi (2020) analyzes the quantity-setting games developed by Poyago-Theotoky (2001) with price-setting games and shows that the result of price-setting games is essentially the same as that of quantity-setting games.

Fjell and Heywood (2004) examine privatization of a public firm in a quantity-setting mixed Stackelberg oligopoly and show that when the public firm remains a leader after privatization, economic welfare will be reduced after privatization. In this paper, we study privatization of a public firm in a price-setting mixed Stackelberg oligopoly.

The remainder of this paper is organized as follows. In Section 2, we describe the basic setting. Section 3 examines the mixed Stackelberg market. Section 4 studies the privatized Stackelberg market. Section 5 compares the result of the mixed Stackelberg market with that of the privatized Stackelberg market. Finally, Section 6 concludes the paper.

## 2. Basic setting

There is an industry composed of a public firm and a private firm producing imperfectly substitutable goods. In the remainder of this paper, subscripts 0 and 1 denote the public firm and the private firm, respectively. In addition, when  $i$  and  $j$  are used to refer to firms in an expression, they should be understood to refer to 0 and 1 with  $i \neq j$ . There is no possibility of entry or exit. The basic setting is taken from Barcena-Ruiz and

Garzón (2007). Firm  $i$ 's demand function is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2} \quad (1)$$

where  $a > 0$ ,  $0 < b < 1$ , and  $p_i$  is firm  $i$ 's price. For the sake of simplicity, we assume  $b = 0.5$ . Each firm's profit is given by

$$\pi_i = (p_i - c + s)q_i \quad (2)$$

where  $c$  represents the total cost for each unit of output and  $s$  is the subsidy for each unit of output. The private firm seeks to maximizes (2). We assume  $0 < c < a$  to assure that the firms' production levels are positive.

Consumer surplus is given by

$$CS = U(q_0, q_1) - p_0q_0 - p_1q_1 \quad (3)$$

where  $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2$ . Economic welfare is

$$W = CS + \pi_0 + \pi_1 - s(q_0 + q_1) \quad (4)$$

The public firm aims to maximizes (4). In this paper, we solve for the subgame perfect equilibrium through backward induction.

### 3. Mixed Stackelberg market

We consider the following three-stage game. In stage one, the government chooses the production subsidy. In stage two, the public firm sets its price. In stage three, the private firm sets its price. Starting from stage three, we obtain

$$p_1^M = \frac{a + 2c - 2s + p_0}{4} \quad (5)$$

where the superscript "M" denotes the value of the mixed Stackelberg duopoly game.

In stage two, the public firm chooses its price for given subsidy anticipating how its choice affects the private firm's price decision. Therefore, we obtain the equilibrium prices in terms of the subsidy:

$$p_0^M = \frac{a + 6c - 2s}{7} \quad (6)$$

$$p_1^M = \frac{2a + 5c - 4s}{7} \quad (7)$$

In stage one, the government anticipating how its choice of subsidy affects firms' price

choices, maximizes (4). The optimal subsidy is

$$\bar{s}^M = \frac{a-c}{2} \quad (8)$$

where the upper bar denotes the equilibrium value. Since  $a > c$ ,  $\bar{s}^M$  is strictly positive.

From (6) – (8), we derive the following subgame perfect equilibrium outcomes:

$$\bar{p}_0^M = \bar{p}_1^M = c$$

$$\bar{q}_0^M = \bar{q}_1^M = \frac{2(a-c)}{3}$$

$$\bar{W}^M = \frac{2(a-c)^2}{3}$$

Note that the public leader's price and output are respectively identical with the private follower's price and output. Also note that each firm sets a price that equals  $c$ .

## 4. Privatized Stackelberg market

In this section, we assume that the public Stackelberg leader is privatized. The private leader decides its price to maximize its profit for a given subsidy anticipating the reaction of the followers as given in (5). This results in

$$p_0^P = \frac{5a + 9c - 9s}{14} \quad (9)$$

where the superscript “P” denotes the value of the privatized Stackelberg duopoly game. Furthermore, we obtain

$$p_1^P = \frac{19a + 37c - 37s}{56} \quad (10)$$

The government optimizes the resulting welfare yielding the following subsidy:

$$\bar{s}^P = \frac{711(a-c)}{1333} \quad (11)$$

From (9) – (11), we can derive the following equilibrium outcomes:

$$\bar{p}_0^P = \frac{19a + 1314c}{1333}$$

$$\bar{p}_1^P = \frac{2701c - 35a}{3999}$$

$$\bar{q}_0^P = \frac{2555(a-c)}{3999}$$

$$\bar{q}_1^P = \frac{2774(a-c)}{3999}$$

$$\bar{W}^P = \frac{5329(a-c)^2}{7998}$$

Note that the optimal subsidy no longer leads to symmetric prices and quantities in equilibrium.

## 5. Comparisons

In this section, we compare the equilibrium values for private leadership with those for public leadership. These comparisons can be depicted as follows:

$$\bar{s}^P - \bar{s}^M = \frac{89(a-c)}{2666}$$

$$\bar{p}_0^P - \bar{p}_0^M = \frac{19(a-c)}{1333}$$

$$\bar{p}_1^P - \bar{p}_1^M = -\frac{35a + 1298c}{3999}$$

$$\bar{q}_0^P - \bar{q}_0^M = -\frac{37(a-c)}{1333}$$

$$\bar{q}_1^P - \bar{q}_1^M = \frac{36(a-c)}{1333}$$

$$\bar{W}^P - \bar{W}^M = \frac{2663(a-c)^2}{7998}$$

The optimal subsidy is higher under privatization. The main result of this study can be summarized in the following proposition.

**Proposition 1:** If privatization results in a public leader becoming a private leader, then the optimal subsidy, the leader's price, the follower's output and economic welfare are higher while the follower's price and the leader's output are lower.

This proposition states that our result makes a sharp contrast with that of quantity-setting market games obtained by Fjell and Heywood (2004).

## 6. Conclusion

We have first considered a price-setting mixed duopoly game when a public firm acts as a leader against a private firm and have shown that the public leader's price and output are respectively identical with the private follower's price and output. Second, we have examined that a price-setting duopoly game when the public firm remains a leader after privatization. Third, we have compared the equilibrium values for private leadership with those for public leadership and have shown that the optimal subsidy and economic welfare are higher in the privatized Stackelberg market than in the mixed Stackelberg market. In consequence, we have found that our result makes a sharp contrast with that of quantity-setting market games.

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